# Independent Research Project Proposal for CS634 Geometric Algorithms under Non-Standard Metrics and for Permutation

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## Overview

The project aims to study the behavior of geometric algorithms when extended beyond classical Euclidean assumptions. Motivated by both in-class material and classical geometric algorithms, I plan to explore the following two directions:

1. Classical geometric structures and algorithms that we learned (e.g., Voronoi diagrams, convex hulls, kd-trees) under alternative distance metrics such as $L_1$, $L_\infty$, or asymmetric divergences.

2. Algorithmic geometry on permutation spaces, where permutations are treated as points under metrics like Kendall tau or Hamming.

To do this, I will also study metric embeddings and dimension reduction as a framework, for unifying and transferring geometric algorithms into these new spaces.

## 1. Classical Geometry in Non-Standard Metrics

Many geometric algorithms assume Euclidean ($L_2$) distance, but their definitions do not depend on this metric. This part of the project will investigate what happens when alternative metrics are used.

For instance, in $L_1$ and $L_\infty$, Voronoi cells become rectilinear polygons (not necessarily convex), and Delaunay triangulations may no longer triangulate the convex hull; also there are data structures like kd-trees aligning well with $L_\infty$ metrics but may misalign with $L_1$ metrics due to orientation. Yet, these graphs may have other desirable properties to explore. I will survey such structures and analyze where they improve or degrade under metric change.

I will also examine whether algorithmic primitives like randomized incremental constructions retain their performance under different metrics, and what geometric challenges appear in the adaptations.

## 2. Geometry on Permutations

Permutations form discrete metric spaces under distances such as Kendall tau, Hamming, and Cayley. These spaces can be viewed geometrically using polytopes such as the permutohedron.
In this direction, I will consider adapting geometric algorithmic tasks, like nearest neighbor search; clustering; or Voronoi-like partitions, to permutation spaces. For instance, how does one define and analyze Voronoi cells in $S_n$ under Kendall tau?
Noticing that Kendall tau embeds isometrically into $\ell_1^{O(n^2)}$, it suggests potential for applying some classical geometric techniques after dimensionality reduction. I want to explore whether some method can be applied effectively to approximate queries in $S_n$.

# Expected Outcomes

- A survey of theoretical results on geometric algorithms in non-Euclidean metrics.
- An investigation of studying geometric problems in permutation spaces.
- An investigation of metric embeddings as a tool to transfer classical algorithms to new settings.
- If feasible, theoretical bounds or sketches of new algorithms.

The following reference list are some classical papers and books I found to begin with.

# References


[1] F. Aurenhammer. Voronoi diagrams–a survey of a fundamental geometric data structure. *ACM Computing Surveys*, 23(3):345–405, 1991.

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[3] M. Charikar. Similarity estimation techniques from rounding algorithms. In *Proceedings of the 34th Annual ACM Symposium on Theory of Computing (STOC)*, pages 380–388, 2002.

[4] M. Deza and T. Huang. Metrics on permutations–a survey. *Journal of Combinatorics, Information & System Sciences*, 23:173–185, 1998.