MATH 430 Fall 2020

Midterm 1

Due by Sep 14th 2020 11:59 PM PST on Gradescope

Instructions: Midterm 1 is open notes and open textbook but closed to internet searches. Calculators are allowed though not needed. You are required to do the exam by yourself and not take the help of any person. Make sure you write down proper steps and reasons, mention any theorem/result you use in your solutions etc. Correct solutions but with unjustified/poorly reasoned claims will not receive full credit.

Answer all questions (5×10 points)

1. Let a, b, N, K all be integers > 1. Given that $gcd(a^N, b^K) = 1$, find $gcd(a, b)$.
   Explain your answer with reasons.

2. Show that 8007 is NOT a sum of three squares. Explain your answer with reasons
   (Hint: Think mod 8)

3. Solve for x where $16x + 10 \cong 46 \mod 37$. Show neat steps

4. Find three consecutive integers, the first of which is divisible by a square, the second by a cube and the third by a fourth power. Show neat steps.

5. For some integer, $n > 1$, you are given than $2^{10} \cong 1 \mod n$ and $3^5 \cong 1 \mod n$. Find a value i where $0 \leq i \leq n - 1$ so that
$$6(2^{2019} + 3^{2019}) \cong i \mod n.$$

   Explain your answer with reasons.