MATH 425a Fake Exam

November 25, 2021

True or False Questions

True or false questions; for each question, please either type 'T' or 'F' in Python. For a statement that is true under some circumstances but false in general, answer 'F'. On the test, **ten** questions will be chosen randomly.

(1) All dense subsets of $\mathbb{R}$ are countable.

(2) The collection of finite length binary strings is countable, whereas the collection of infinite length binary strings is uncountable.

(3) Every norm induces a metric, and every metric induces a norm.

(4) Under the notions of addition and multiplication, $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$ are not fields but $\mathbb{R}$ and $\mathbb{C}$ are.

(5) An arbitrary union of closed sets is closed.

(6) The only compact subsets of $\mathbb{R}$ are intervals of form $[a, b]$, where $a < b$ and both are finite.

(7) Fix a rational number q. Then

$$\limsup_{n\to\infty}(nq - \lfloor nq \rfloor) < 1.$$

(8) Fix an irrational number r. Then

$$\limsup_{n\to\infty}(rn - \lfloor rn \rfloor) = 1.$$

(9) Let $\{a_n\}$ be a sequence. If $\limsup\limits_{n\to\infty}|a_{n+1}/a_n| \leqslant 1$ then $\sum a_n$ converges absolutely.

(10) A closed and bounded subset of $\mathbb{R}^n$ is compact. (Here we are using the standard Euclidean metric.)

(11) In a complete metric space, a set is compact if and only if it is bounded.

(12) The continuous image of a open set is open.

(13) The continuous image of a compact set is compact.

(14) The continuous image of a connected set is connected.

(15) The continuous image of a disconnected set is disconnected.

(16) If f is differentiable at x and $f'(x) > 0$, then f is locally increasing.

(17) If a function is Riemann-Stieltjes integrable then it is Riemann integrable.

(18) Uniform convergence is equivalent to convergent in sup norm.

(19) If $f_n \to f$ pointwise, and f_n, f are all continuous, then $f_n \to f$ uniformly.

(20) If a function is continuous, it must be differentiable at at least one point.

Fill-in-the-blank Questions

Try to fill in the blanks as accurately as possible. The grades will be computed automatically, but rooms for typos will be given: for e.g., if the correct answer is “`finite-dimensional`” and you wrote “`finite demensional`”, you will still receive full credit, but if you typed “`banana%#&`” you will of course not receive any points!
The answer for each question consists of *exactly* one word. One of them includes a hyphen. All letters should be in lowercase.

(21) A function is continuous if the ____ (noun) of any closed set is closed.

(22) A set E is ____ (adjective) if it is closed and has no isolated point; that is, every point in E is a limit point.

(23) The limsup of a sequence is the supremum of all subsequential ____ (noun, plural).

(24) A sequence ____ (verb) if and only if its limsup equals its liminf.

(25) A sequence that converges but diverges absolutely is said to converge ____ (adverb).

(26) If f_n converges to f ____ (adverb) and each f_n is continuous, then f is continuous.

(27) The Weierstraß Approximation Theorem states that (Bernstein) ____ (noun, plural) are dense in $C([0, 1])$.

(28) Banach Contraction Principle: in a Banach space, a strong contraction has a unique ____ (noun).

(29) The Devil’s staircase / Cantor function is differentiable almost everywhere; wherever the derivative exists, it equals ____ (a English word).

(30) The Weierstraß Monster is ____ (answer something other than ‘`not`’) differentiable.