> **Remark.** For simplicity of notation, I will denote *permutation* and *combination* of n choose r by
> $$P_n^r := \frac{n!}{(n-r)!} \text{ and } C_n^r := \frac{n!}{r!(n-r)!}, \text{ respectively.}$$

Chapter 1 Problems

Prob.1.8 If a word of length n has no repeating letters then the answer is simply $n!$. If, say, a letter appears twice in the word, then one should divide the factorial by 2! because order of identical letters doesn't matter. Now back to the question: *Fluke* simply has $5! = 120$ arrangements. *Purpose* has a pair of p and a pair of o so the answer is $7!/(2!2!) = 1260$. *Mississippi* has four i's, four s's, and a pair of p. It's total length is 11. The answer is therefore $11!/(4!4!2!) = 34650$. *Arrange* is the same as *Purpose*.

Prob.1.10

(a) Simply $8! = 40320$.

(b) There are 14 ways to arrange these two people. The other 6 can sit freely once A and B have sat down. The answer is $14 \cdot 6! = 10080$.

(c) If men sit at odd numbers then we have $4! \cdot 4!$ arrangements. If women sit at odd numbers we have another $4! \cdot 4!$. Thus the total number is $2 \cdot 4! \cdot 4! = 1152$.

(d) $5! = 120$ ways to arrange the 5 men. Then the rest can sit freely. $5! \cdot 3! = 720$.

(e) Treating each couple as a "block" first, we have 4 blocks and 24 arrangements for these blocks. Within each block there are 2 possibilities. $24 \cdot 2^4 = 384$.

Prob.1.11

(a) $6! = 720$.

(b) Use the "block" strategy again: $3! \cdot (1!2!3!) = 72$.

(c) Still the "block" strategy but with 4 blocks (one for novels, one for each remaining book): $4!3! = 144$.

Prob.1.15 First we choose these people: $C_{10}^5 \cdot C_{12}^5$ options. Then to pair them off we have 5! options. Thus the total number is $C_{10}^5 \cdot C_{12}^5 \cdot 5!$.

Prob.1.23 $3! \cdot 2^3 = 48$.

Prob.1.26 This is a terrible amount of computation...

$$\begin{aligned}(x_1 + 2x_2 + 3x_3)^4 &= \sum_{i+j+k=4} \binom{4}{i,j,k} x_1^i (2x_2)^j (3x_3)^k \\ &= x_1^4 + 8x_1^3x_2 + 12x_1^3x_3 + 24x_1^2x_2^2 + 72x_1^2x_2x_3 + 54x_1^2x_3^2 \\ &\quad + 32x_1x_2^3 + 144x_1x_2^2x_3 + 216x_1x_2x_3^2 + 108x_1x_3^3 + 16x_2^4 + 96x_2^3x_3 + 216x_2^2x_3^2 + 216x_2x_3^3 + 81x_3^4.\end{aligned}$$

Prob.1.33

(a) This is the same thing if we subtract all the requirements from 20000 and divide everything by 1000 just for simplicity. Then we have 9 dollars and we need to invest or not invest them in 4 opportunities. This is then equivalent to investing 13 dollars in 4 opportunities, each having at least 1 dollar invested. By stars-n-bars the answer is $C_{12}^{3} = 220$.

(b) Similar to above, except we analyze case by case. Without the first opportunity, it's C_{13}^{2}. Same for the second one. Without the third and fourth, the possibilities are C_{14}^{2} and C_{15}^{2}, respectively. Adding them together,

$$C_{12}^{2} + C_{15}^{2} + C_{14}^{2} + C_{13}^{2} + C_{13}^{2} = 572 \text{ possibilities.}$$