Beginning of April 5, 2021

Proposition 0.0.1

If such T exists, it is unique.

Proof. If T and S both satisfy the limit condition, then

$$\lim_{v\to\infty} \frac{S(v) - T(v)}{\|v\|} = 0 = \lim_{v\to\infty} \frac{(S-T)(v)}{\|V\|}$$

by directly subtracting one from another. Assume $S - T$ is <u>not</u> the zero linear transformation, so there exists $v_0 \neq 0$ such that $(S-T)(v_0) \neq 0$. On one hand, we should have $\lim\limits_{c\to\infty} cv_0 = 0$, whereas

$$\lim_{c\to 0+} \frac{(S-T)(cv_0)}{\|cv_0\|} = \frac{(S-T)(v_0)}{\|v_0\|}$$

□