

# Math 425b

## Fundamental Concepts of Analysis

**Section / Session:** 39647R, 060

**Instructor:** Andy Manion

**Office Hours:** To be announced

**Office Hours Location:** Usual lecture Zoom room

**Email:** [amanion@usc.edu](mailto:amanion@usc.edu)

**Teaching Assistant:** To be announced

**Units:** 4.0

**Term:** Spring 2021

**Day:** MWF

**Time:** 9:00 AM (Pacific)

**Location:** Online

### Course Description

In this course, we will continue studying the rigorous underpinnings of calculus from a modern perspective, as started in Math 425a. After finishing up Riemann integration and series, we will consider notions of convergence of sequences and series of functions, especially uniform convergence, and prove the Arzelà–Ascoli theorem and the Stone–Weierstrass theorem. Then we will focus on two major applications of the contraction mapping principle for complete metric spaces, namely Picard's existence/uniqueness theorem for ODEs and the inverse/implicit function theorems in multivariable calculus. Our perspective on multivariable calculus will emphasize the Jacobian or total derivative of a function as a linear map. Finally, we will study multivariable calculus from the perspective of differential forms on  $\mathbb{R}^n$ , including the relevant linear algebra (tensor products, dual spaces, etc.).

### Prerequisite(s): Math 425a

This course is best suited for students who have taken 425a and are ready to build on their knowledge from that course, or students with equivalent preparation from other sources. Familiarity with the basic point-set topology of metric spaces, covered in 425a, is an important prerequisite. The course will be proof-based and some familiarity and fluency with proofs will be required; the course should also be a good opportunity to practice and improve at reading and writing mathematical proofs.

## Learning Objectives

- Reproduce definitions and proofs for classical results on series, uniform convergence, differential equations, multivariable calculus, and multilinear algebra / differential forms
- Evaluate whether statements on these topics are correct
- Develop and write proofs for unfamiliar statements on these topics; find counterexamples when the statements are false
- Perform basic computations with differential forms on  $\mathbb{R}^n$ , including integration, wedge products, pullbacks, and exterior derivatives

## Required Readings and Supplementary Materials

*Real Mathematical Analysis*, Charles C. Pugh, Springer, second edition; available at the USC bookstore.

Pugh's book will be the only required textbook for the course; later in the course there will be some material on differential forms integrated into the homework assignments. Reading assignments will be given to indicate how far you should read in the textbook to keep up with lecture; I recommend completing all reading assignments before the corresponding lectures to maximize your ability to follow along. This also helps you ask questions about things that were confusing in the book when the corresponding material arises in lecture.

## Course Schedule

Time permitting, we will cover Sections 3.3, 4.1-4.5 (and maybe 4.6), 5.1-5.4, and 5.7-5.9 of Pugh's book (Sections 5.8 and 5.9 may possibly be replaced or supplemented by an alternative set of notes). This rough guess may be added to or subtracted from, based on how fast we end up going.

## Grading Breakdown

Letter grade. Your grade will be based on your scores in the following categories:

Category	Number of Scores	% of Grade
Homework	13	35
Midterm exams	2	40 (20 each)
Final exam	1	25
<b>TOTAL</b>		<b>100%</b>

*Your lowest two homework scores will be dropped from your grade*, giving you some leeway to miss assignments in emergencies. This policy is not intended to be used strategically; plan to do all your homework assignments and only make use of this policy in rare circumstances.

## Assessment of Assignments and Exams

All homework assignments and exams will be graded on correctness, completeness, clarity, and concision. Here are some additional notes:

- You must write in full sentences with correct grammar, with the usual allowances for mathematical notation and close-enough grammar.
- Just “having the right idea” is not enough. Every sentence of your proof needs to be true and relevant to your argument.
- Your proof needs to be complete, in that no essential steps are missing. However, there is a balance in what to include versus what to leave out; try to follow the book’s model in writing your proofs. Make sure that you don’t introduce irrelevant claims; write only what you need for the proof.

## Assignment Submission Policy

Reading assignments should be completed by the date indicated on the assignment. Problem sets are due on Fridays by 11:59 PM (Pacific) starting Friday, Jan. 22; they should be submitted on Gradescope. We will try to grade and return them by the following Friday.

Try to avoid late submission of homework; please ask me before the deadline if you need an extension, or we may take points off or not count the assignment.

## Exams

There will be two midterm exams scheduled for Friday, February 26 and Friday, April 2, as well as a final exam scheduled for Friday, May 7 from 8-10 am (Pacific). My tentative plan is to give take-home, closed-book exams on the honor system, since (given academic integrity) this type of exam is a good substitute for the usual in-person experience of 425b while hopefully permitting flexibility with your schedules. If we do this type of exam, there will be at least a few-day window around the midterm exam dates and you will be free to choose a continuous period of time (a few hours probably) during those few days to take the exam. For the final, we’ll probably aim to have all the exams completed by 10am Pacific on Friday, May 7.

## Grade appeals

You should submit any questions on exam grading to me by email; I will consult with the TAs and see if any grading mistakes were made. The deadline for submitting regrade requests is *two weeks* after an exam; for a Friday exam, all regrade requests must be submitted by the end of the day two Fridays later. If you are still unsatisfied with a grading decision after I have reviewed it, there is a two-level appeals process described in <https://policy.usc.edu/scampus-part-c/>.

## Missing homework scores

All inquiries about missing homework scores are also subject to a two-week deadline. If your score is missing on Blackboard, or you did not get your assignment returned to you, you should notify me by email by two weeks after the assignment deadline. Please keep up with your scores and make sure they’re accurate throughout the semester instead of waiting until the end of the semester to check for missing scores.

## **Technological Proficiency and Hardware/Software Required**

The required technology in this course will be Zoom, Blackboard, and Gradescope as well as some sort of scanner (e.g. a smartphone) for uploading assignments and exams to Gradescope. More details will be forthcoming, especially on Gradescope; if you have any difficulties with course technology, just ask me.

LaTeX is recommended but not required for homework assignments. To use LaTeX, you need a LaTeX distribution (I recommend MiKTeX <https://miktex.org/download> or TeX Live <https://www.tug.org/texlive/> for Windows, MacTeX <https://tug.org/mactex/> for Mac, and TeX Live for Linux). You also need a text editor and there are various options with LaTeX integration; I use TeXnicCenter <http://www.texniccenter.org/> for Windows, and MacTeX has its own editor included. TeXworks <https://www.tug.org/texworks/> is another option for all three platforms, and many Windows users like WinEdt <https://www.winedt.com/>. All of these programs are free to download and use.

## **Additional Policies**

### **Students with Disabilities**

Any student requesting academic accommodations based on a disability is required to register with Disability Services and Programs (DSP) each semester. A letter of verification for approved accommodations can be obtained from DSP. Please be sure the letter is delivered to me (or to the TA) as early in the semester as possible. DSP is located in GFS 120 and is open 8:30 a.m.–5:00 p.m., Monday through Friday.

- Website for DSP: <https://dsp.usc.edu/>
- Contact information: (213) 740-0776 (Phone), (213) 740-6948 (TDD only), (213) 740-8216 (FAX)
- Email for DSP: [dspfrontdesk@usc.edu](mailto:dspfrontdesk@usc.edu).

## **Statement on Academic Conduct and Support Systems**

### **Academic Conduct**

Plagiarism — presenting someone else's ideas as your own, either verbatim or recast in your own words — is a serious academic offense with serious consequences. Please familiarize yourself with the discussion of plagiarism in *SCampus* in Part B, Section 11, *Behavior Violating University Standards* <https://policy.usc.edu/scampus-part-b>. Other forms of academic dishonesty are equally unacceptable. See additional information in *SCampus* and university policies on scientific misconduct, <http://policy.usc.edu/scientific-misconduct/>.

### **Support Systems**

- Counseling and Mental Health - (213) 740-9355 – 24/7 on call  
Free and confidential mental health treatment for students, including short-term psychotherapy, group counseling, stress fitness workshops, and crisis intervention.  
[studenthealth.usc.edu/counseling](http://studenthealth.usc.edu/counseling)

- National Suicide Prevention Lifeline – 1 (800) 273-8255 – 24/7 on call  
Free and confidential emotional support to people in suicidal crisis or emotional distress 24 hours a day, 7 days a week.  
[www.suicidepreventionlifeline.org](http://www.suicidepreventionlifeline.org)
- Relationship and Sexual Violence Prevention and Services (RSVP) - (213) 740-9355(WELL), press “0” after hours – 24/7 on call  
Free and confidential therapy services, workshops, and training for situations related to gender-based harm.  
[studenthealth.usc.edu/sexual-assault](http://studenthealth.usc.edu/sexual-assault)
- Office of Equity and Diversity (OED)- (213) 740-5086 — Title IX – (213) 821-8298  
Information about how to get help or help someone affected by harassment or discrimination, rights of protected classes, reporting options, and additional resources for students, faculty, staff, visitors, and applicants. The university prohibits discrimination or harassment based on the following protected characteristics: race, color, national origin, ancestry, religion, sex, gender, gender identity, gender expression, sexual orientation, age, physical disability, medical condition, mental disability, marital status, pregnancy, veteran status, genetic information, and any other characteristic which may be specified in applicable laws and governmental regulations. The university also prohibits sexual assault, non-consensual sexual contact, sexual misconduct, intimate partner violence, stalking, malicious dissuasion, retaliation, and violation of interim measures.  
[equity.usc.edu](http://equity.usc.edu), [titleix.usc.edu](http://titleix.usc.edu)
- Reporting Incidents of Bias or Harassment - (213) 740-5086 or (213) 821-8298  
Avenue to report incidents of bias, hate crimes, and microaggressions to the Office of Equity and Diversity —Title IX for appropriate investigation, supportive measures, and response.  
[usc-advocate.symplicity.com/care\\_report](http://usc-advocate.symplicity.com/care_report)
- The Office of Disability Services and Programs - (213) 740-0776  
Support and accommodations for students with disabilities. Services include assistance in providing readers/notetakers/interpreters, special accommodations for test taking needs, assistance with architectural barriers, assistive technology, and support for individual needs.  
[dsp.usc.edu](http://dsp.usc.edu)
- Campus Support & Intervention - (213) 821-4710  
Assists students and families in resolving complex personal, financial, and academic issues adversely affecting their success as a student.  
[campussupport.usc.edu](http://campussupport.usc.edu)
- Diversity at USC - (213) 740-2101  
Information on events, programs and training, the Provost’s Diversity and Inclusion Council, Diversity Liaisons for each academic school, chronology, participation, and various resources for students.  
[diversity.usc.edu](http://diversity.usc.edu)
- USC Emergency - UPC: (213) 740-4321, HSC: (323) 442-1000 – 24/7 on call  
Emergency assistance and avenue to report a crime. Latest updates regarding safety, including ways in which instruction will be continued if an officially declared emergency makes travel to campus infeasible.  
[dps.usc.edu](http://dps.usc.edu), [emergency.usc.edu](http://emergency.usc.edu)
- USC Department of Public Safety - UPC: (213) 740-6000, HSC: (323) 442-120 – 24/7 on call  
Non-emergency assistance or information.  
[dps.usc.edu](http://dps.usc.edu)