Jiaming Shi MATH 425a midterm 1

1\. $\sup A = \infty$, $\inf A = -\frac{5}{2}$ (for sup, fix $n=-2$, $m$ can be $-\infty$; for inf, fix $m$ ~~positive~~ 5, $n$ -2)

2\. A not countable ~~I (0,1) is a subset I~~, B countable (there is a bijection of $N \to (a,b)$ since $a, b \in \mathbb{Q}$)

3\. a) Neither.

b) $N$ is closed, $(0,1]$ is not.

c) $[0,1]$ is perfect, $N$ is not.

d) Neither.

4\. a) Say ~~[-0.5, 0]~~ $\{(x,y) \in R^2 \mid$ ~~[illegible]~~ $x \in [-0.5, 0], y = 0\} \cup \{(0.1, 0)\}$.

b) Closure is $E \cup E'$. In this case, $B_0$'s closure is a circle of radius 1, center $(0,0)$ in $R^2$ ($x^2+y^2 \le 1$).

c) No, consider point $(0,1)$ for example. No $\varepsilon > 0$ satisfies $N_\varepsilon((0,1)) \subset X$.

5\. No. This is because Heine-Borel theorem states that a ~~set~~ subset of $R^n$ is compact if and only if it's BOTH closed and bounded. If $K$ is only closed, this does not imply sequential compactness.

6\. a) Yes.

b) $E := $ ~~[illegible]~~ $\{1\}$

c) Yes.

d) Yes.

7\. ~~[illegible]~~ $\liminf_{n\to\infty} x_n = -1$, $\limsup_{n\to\infty} x_n = 1$

8\. Proof: we first verify that this is true for $n=1$. $1 \cdot 2^1 = 2 = 2 + (1-1)2^{1+1} = 2$. LHS = RHS.

Suppose the argument is true for $n$. Then $1\cdot 2^1 + 2\cdot 2^2 + \cdots + n \cdot 2^n = 2 + (n-1)2^{n+1}$.

$1 + 2^1 + 2^2 + \cdots + n \cdot 2^n + (n+1)2^{n+1} = 2 + (n-1)2^{n+1} + (n+1)2^{n+1} = 2 + 2^{n+1}(2n) = 2 + n \cdot 2^{n+2}$.

↳ we thus conclude that this argument is true for $n+1$ when true for $n$.

Therefore, this argument is true $\forall n \in N$.

9\. ~~If R is compact, in a sequence $\{(x-r_n, x+r_n)\}_{n\ge1}$ (let this be $\{y_n\}$) we can extract a subsequence $\{y_{n_k}\}$ that converges in $x \in R$ as $n\to\infty$~~

If $R$ is compact, then from $\{(x-r_n, x+r_n)\}_{n\ge1}$, we can extract finitely many subcovers that cover this collection of open interval. However, we know for $\{r_n\}_{n\ge1}$, $r_n \to \infty$ as $n \to \infty$. This means we need infinitely many open subcovers to cover the interval when $n$ is arbitrarily large. Hence $R$ is not compact because this interval is a subset of $R$.

10\. We know ∴ $0 \le$ ~~lim~~ $(1+\frac{2}{n})^n \le e$.

$\because 0 \cdot (\frac{1}{n}) \le \frac{1}{n}(1+\frac{2}{n})^n \le e(\frac{1}{n})$, $\lim_{n\to\infty} 0 = \lim_{n\to\infty} \frac{e}{n} = 0$,

$\therefore \lim_{n\to\infty} x_n = \lim_{n\to\infty} \frac{1}{n}(1+\frac{2}{n})^n = 0$.