

# **MATH 507a**

## **Fall 2022**

**Professor** Ken Alexander

**Phone:** 213-740-3797

**Office:** KAP 424E. It is inside suite 424, in the corner of the building that's behind you and to your right as you exit the elevator.

**Email:** alexandr@usc.edu      **Web Page:** <https://dornsife.usc.edu/kenneth-alexander/> . Links on this page will take you to course information and homework assignments.

**Office hours:** Monday 2-3, Wednesday 10-12. For an appointment outside office hours, it's best to call or email at least 1-2 hours ahead. (No appointment necessary for office hours.)

**Grader:** Henry Ehrhard

**Text:** *Probability: Theory and Examples*, 5th edition, by R. Durrett. The 5th<sup>h</sup> edition has not been published yet, but for several years it's been available at Rick Durrett's web site: [https://services.math.duke.edu/~rtd/PTE/PTE5\\_011119.pdf](https://services.math.duke.edu/~rtd/PTE/PTE5_011119.pdf) .

**Prerequisite:** Math 525a (graduate real analysis) is listed as a prerequisite, but the courses can be taken at the same time if you had a strong undergraduate analysis course.

**Homework** will be given approximately weekly. On each assignment a few selected problems will be graded. It is OK to help each other solve homework problems, but it's not OK to turn in essentially identical solutions--once you have discussed homework problems so that you understand better what to do, you should write the solutions on your own. Homework will be accepted up to 2 lectures late, with a 20% penalty. Not accepted more than 2 lectures late. The deadline for both on-time and late homework is the end of the lecture hour (not the end of the day.) Homework should be turned in either (i) in lecture, or if that's not possible, (ii) by sliding under my office door. *Do not place homework directly in my mailbox!*

**Exams:** One midterm plus final. Tentative midterm date is Wednesday October 12. Final has both take-home and in-class components. In-class final is Wednesday December 14, 11 am – 1 pm. The final is cumulative but with greater emphasis on material after the midterm. Exam locations to be announced.

**Missed exams:** There will be no makeup exams. In exceptional cases, with a written excuse, if you miss an exam I will base your grade on the exams you did take.

**Grading:** Homework 35%, midterm 25%, final 40% (= about 15% from take-home and 25% from in-class)

**Auditing:** If you want to just sit in on the class, you must register for an audit.

Everything in this syllabus is in principle tentative; if necessary it will be adjusted during the semester.

**Statement for Students with Disabilities:** Any student requesting academic accommodations based on a disability is required to register with Disability Services and Programs (DSP) each semester. A letter of verification for approved accommodations can be obtained from DSP. Please be sure the letter is delivered to me (or to TA) as early in the semester as possible. DSP is located in STU 301 and is open 8:30 a.m.–5:00 p.m., Monday through Friday. The phone number for DSP is (213) 740-0776.

## **Statement on Academic Conduct and Support Systems**

**Academic Conduct:** Plagiarism – presenting someone else’s ideas as your own, either verbatim or recast in your own words – is a serious academic offense with serious consequences. Please familiarize yourself with the discussion of plagiarism in *SCampus* in Section 11, *Behavior Violating University Standards*<https://scampus.usc.edu/1100-behavior-violating-university-standards-and-appropriate-sanctions/>. Other forms of academic dishonesty are equally unacceptable. See additional information in *SCampus* and university policies on scientific misconduct, <http://policy.usc.edu/scientific-misconduct/>.

Discrimination, sexual assault, and harassment are not tolerated by the university. You are encouraged to report any incidents to the Office of Equity and Diversity <http://equity.usc.edu/> or to the Department of Public Safety <http://capsnet.usc.edu/department/department-public-safety/online-forms/contact-us>. This is important for the safety whole USC community. Another member of the university community – such as a friend, classmate, advisor, or faculty member – can help initiate the report, or can initiate the report on behalf of another person. The Center for Women and Men <http://www.usc.edu/student-affairs/cwm>provides 24/7 confidential support, and the sexual assault resource center webpage [sarc@usc.edu](mailto:sarc@usc.edu) describes reporting options and other resources.

**Approximate schedule:**

Lectures 1-3: sigma-fields, probability measures, distribution functions, measure theory review

Lectures 4-6: random variables, distributions, absolute continuity, measurability

Lectures 7-9: integration, expectation, convergence theorems for expectation, inequalities, convergence of moments

Lectures 10-11: independence, joint distributions, Kolmogorov extension theorem, Fubini theorem

Lectures 12-13: weak laws of large numbers, covariance

Lectures 14-15: Borel-Cantelli Lemma and applications, Kolmogorov 0-1 law

Lectures 16-17: Kolmogorov inequalities, 3 series theorem, more inequalities

Lectures 18-19: strong law of large numbers, renewal theory, Glivenko-Cantelli theorem

Lecture 20: large deviations

Lectures 21, 23-25: weak convergence, a.s. convergent realizations, other modes of convergence, tightness, Helly selection theorem

Lecture 22: Exam

Lectures 26-30: characteristic functions and applications

Lectures 31-33: central limit theorem, Lindeberg-Feller theorem, normal approximation, Poisson convergence

Lectures 34-36: limit theorems in  $\mathbb{R}^d$ , multivariate normal

Lectures 37-38: conditional expectation, Radon-Nikodym theorem, inequalities,  $L^2$  space of random variables

Lectures 39-41: martingales, decomposition, convergence, applications.