&

f

$D_r(a)$

$f^{(n)}(a) =$

0

$n \in$

N

0

$f \equiv$

0

$D_r(a)$

Intuition: if we know existence of Taylor series then this is obvious. But we don't.

f

$\overline{D_r(a)}$

$M := \sup_{\overline{D_r(a)}} f$

n

f_n

$D_r(a)$

$$f(z) = f_n(z)(z-a)^n + \sum_{j=0}^{n-1} \frac{f^{(j)}(a)}{j!}(z-a)^j = f_n(z)(z-a)^n.$$

$$f_n(z) = \frac{1}{2\pi i} \int_{\partial D_r(a)} \frac{f(\zeta)}{(\zeta - a)^n (\zeta - z)} \, \mathrm{d}\zeta.$$

${}_n(z) \frac{2\pi r \sup f}{2\pi r^n \cdot (r-z-a)} =$

$\frac{M}{r^{n-1}(r-z-a)}$

$\frac{Mz-a^n}{r^{n-1}(r-z-a)}.$

$z-$

$a <$

r

$n \to$

∞

$f_n(z) \to$

0

$f \equiv$

0

f

$\Omega \subset$

C

$a \in$

Ω

$r >$

0

$\overline{D_r(a)} \subset$

Ω

$$f(z) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(z-a)^n for all z \in D_r(a).$$

$R(z) =$

$f_n(z)(z-$

$a)^n$

0

$R = \sup\{r > 0 : \overline{D_r(a)} \subset \Omega\}.$

f

$\Omega \subset$

C

connected

$f^{(n)}(a) = 0 for all n \in N \cup \{0\} *$

some

$a \in$

Ω

$f =$