

PHIL236 HW #1

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Using the notions of congruence, betweenness and point, and the definition of a region as a collection of points, define the following notions.

- (1) The notion of a straight line (finite, infinite, closed, or open).

Solution. A straight line is a collection \mathcal{C} of points satisfying the following:

- (Colinearity) For all $x, y, z \in \mathcal{C}$, there exists permutation σ on $\{x, y, z\}$ such that $\text{Bet}(\sigma(x), \sigma(y), \sigma(z))$.
- (Continuum) For all $x, z \in \mathcal{C}$ and any $y \in \Omega$ the universal space satisfying $\text{Bet}(x, y, z)$, we have $y \in \mathcal{C}$.

- (2) The notion of a straight line that is infinite in both directions.

Solution. A doubly infinite straight line is a straight line also satisfying the following additional criterion:

- (Unboundedness) For all $x, y \in \mathcal{C}$, there exists $z \in \mathcal{C}$ such that $xy \equiv yz$ (congruent) and $x \neq z$.

- (3) “ xy is at least as long as zw .”

Solution. There exists u such that $\text{Bet}(x, u, y)$ and $xu \equiv zw$. (We assume that $\text{Bet}(x, x, y)$ and $\text{Bet}(x, y, y)$ both trivially hold.)

- (4) In proposition 1 of the elements, Euclid constructs a pair of congruent circles each of whose center lies on the edge of the other. He then shows, from his basic postulates, that the centers of these two circles, along with one of the points at which the circles intersect is an equilateral triangle (a triangle with equal sides). How does Euclid know that there is a point at which the circles intersect? Do you think this can be proven from his postulates?

Solution. No, it seems like, and according to the footnote in the text, that two more implicit assumptions have been made: (i) two circles constructed as such intersect, and (ii) straight lines (CA, CB) intersect at at most one point.