PHIL 236 Homework (1/19)

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Using the notions of congruence, betweenness and point, and the definition of a region as a collection of points, define the following.

(1) **Sphere.**

Solution. A sphere is a collection $\mathcal{S}$ of points such that, there exists a point p, with $px \cong py$ (congruent) for all $x, y \in \mathcal{S}$. Further, there does not exist points outside $\mathcal{S}$ satisfying this congruence. In fact such p is unique if it exists, and we call it the *center*.

(2) **Plane.**

Solution. Let three non-colinear points, x, y, z, be given. We define the *triangle* characterized by these three points as the union of three finite closed line segments with endpoints chosen pairwise from $\{x, y, z\}$ and denote it by Δxyz. A plane characterized by $\{x, y, z\}$, is the collection $\mathcal{P}$ of points p satisfying:

$$\text{there exists a line } \ell, p \in \ell, \text{ such that } |\ell \cap \Delta xyz| > 1.$$

(3) **Circle.**

Solution. A collection of points is a circle if and only if it is the nonempty, non-singleton intersection of some sphere with some plane. (It depends on whether we consider empty sets or singletons as trivial circles.)

(4) **Three points, x, y, z, forming a right angle (at y).**

Solution. Let ℓ be the doubly infinite line containing y and z. we say y is a right angle if for all $w \in \ell$, xw is at least as long as xy (using previous HW).

(5) **Parallel lines.**

Solution. Two lines ℓ_1, ℓ_2 are parallel if the following are satisfied:

- (Co-planar) There exists a plane $\mathcal{P}$ such that $\ell_1 \subset \mathcal{P}$ and $\ell_2 \subset \mathcal{P}$.
- (Non-intersecting) (Extend ℓ_1, ℓ_2 to be doubly infinite, if needed, using previous HW.) $\ell_1 \cap \ell_2 = \varnothing$.