

PHIL 236 Homework (2/5)

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Problem 1

Let us say that three balls in ordinary Euclidean space are “right angled” if and only if their centers are at right angles. Does this notion make sense in a non-Euclidean pointless geometry, where there are no points, and thus no centers?

Solution. This notion still makes sense, since in class we’ve derived what it means to be tangent, diametrically opposite (an equivalent notion of *betweenness*), concentric, and congruent (equivalent to congruence in Euclidean geometry also), in that order, in a pointless geometry. Since we were able to define right angles using congruence and betweenness only (e.g. introducing a fourth point colinear with two current ones such that a series of congruence relationships are satisfied), we can achieve the equivalent definition under a pointless geometry using our previous derivations.

Problem 2

In Euclidean geometry, do you think it is possible to define the relation of x being 1 meter from y using congruence and betweenness only?

Solution. No. Both notions are qualitative whereas “ x being 1 meter from y ” is quantitative. (Alternatively, these two notions are invariant under *similarity transformations* whereas the statement isn’t.)

Problem 3

Can you define the primitive of Kantian geometry — w is on the left of the plane defined by xyz — using only congruence and betweenness?

Solution. No. Both congruence and betweenness are symmetric, i.e., $\text{Bet}(x, y, z) \Leftrightarrow \text{Bet}(z, y, x)$ and $xy \cong zw \Leftrightarrow yx \cong zw$. Therefore, if we somehow designated one side of a plane to be its “left” side, we can simply flip the order of all notations in congruence and betweenness and consequently arrive at the funny conclusion that the other side is the “left” side, without changing any assumptions. (Alternatively, apply a *similarity transformation* and use the same argument.)